MIXED CONVECTION AROUND A CIRCULAR CYLINDER IN A HELE-SHAW CELL

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The well-known Hele-Shaw cell consists of two closely spaced parallel plates, the space between which is filled with fluid. The dimensions of the cell in the plane of the boundary plates are much greater than the distance between them – the cell thickness. At low velocities the flow in the Hele-Shaw cell models two-dimensional potential flow and is the hydrodynamic analog of single-phase flow through a porous medium [1]. This analogy is based on the coincidence of the corresponding equations of motion (Darcy's law). In a number of important technical applications, in which flat-channel flow is encountered, the flow may be high- as well as low-velocity.

One such application is the flow in hydraulically fractured rocks, one of the most promising methods of creating closed circulation systems for extracting geothermal energy [2]. Whereas at a distance from the boreholes creep flow is realized, as the borehole is approached inertial effects become increasingly apparent. In these circumstances not only the ideal case of symmetric radial flow but also nonsymmetric jet flow and flow past impermeable inclusions may be realized. Jet flow and flow past a cylinder in a flat channel were investigated in [3-5].

Modern high-speed highly integrated computer circuits are characterized by high heat release (~10^2 W/cm^2). Boards with built-in integrated logic elements are cooled by immersion in a coolant flow [6]. In this case, if the boards are closely spaced, flow in a flat channel of variable thickness is realized.

A solution is proposed for the stationary problem of laminar viscous incompressible flow past a circular cylinder with internal heat sources in a Hele-Shaw cell with thermally insulated walls (the generators of the cylinder are perpendicular to the planes of the cell). The problem models the flow past a heat-release element in a flat channel. For simplification purposes it is assumed that the height of the cylinder is equal to the thickness of the cell and the problem is considered in the two-dimensional approximation.

Confining ourselves to the case of moderate temperature differences in the fluid, we will base our analysis of the problem on the classical Oberbeck-Boussinesq model:

$$(\mathbf{U}\cdot\nabla)\mathbf{U} + (1/\rho)\nabla p = \nu\Delta\mathbf{U} - \mathbf{g}\beta(T - T_\infty),\ (\mathbf{U}\cdot\nabla)T = a\Delta T,\ \operatorname{div}\mathbf{U} = 0 \tag{1}$$

($\mathbf{U}$ and $\mathbf{g}$ are the velocity and gravity vectors, p is the pressure, T is the temperature of the fluid, T_∞ is the approach stream temperature at infinity, ρ is the density, ν is the kinematic viscosity, a is the coefficient of thermal diffusivity, and β is the volume coefficient of expansion of the fluid).

On the flat surfaces of the cell $\{z = h/2\}$ and $\{z = -h/2\}$ and on the lateral surface of the cylinder the no-slip conditions are satisfied. At an infinite distance from the cylinder $\mathbf{U} = (\bar{U}_0(1 - 4z^2/h^2), 0, 0)$.

The thermal problem is solved for boundary conditions of the first kind: $T = T_0$ on the cylinder wall (T_0 is the constant temperature of the lateral surface of the cylinder) and $T = T_\infty$ at an infinite distance from the lateral surface of the cylinder. The cell is arranged vertically with the z component of the gravity vector $g_z = 0$.

We assume that throughout the flow domain the fluid temperature is independent of the z coordinate, there is no motion of the fluid in the direction of the z axis, and for the longitudinal and transverse velocity components we have a Poiseuille profile:

$$u(x, y, z) = u_0(x, y)(1 - 4z^2/h^2),\ v(x, y, z) = v_0(x, y)(1 - 4z^2/h^2).$$

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Integrating the equations of system (1) with respect to z from 0 to h/2, on the assumption that the walls of the cell are thermally insulated, we obtain the following system of equations for the velocity $\mathbf{U}_0 = (u_0, v_0)$ in the plane of symmetry of the cell, the pressure p and the temperature T:

$$(\mathbf{U}_0\cdot\nabla)\mathbf{U}_0 + \frac{15}{8\rho}\nabla p = \frac{5}{4}\nu\left(\frac{\partial^2\mathbf{U}_0}{\partial x^2} + \frac{\partial^2\mathbf{U}_0}{\partial y^2}\right) - \frac{15}{8}g\beta(T - T_\infty) - 15\frac{\nu}{h^2}\mathbf{U}_0,$$
$$(\mathbf{U}_0\cdot\nabla)T = \frac{3}{2}a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right),\quad \operatorname{div}\mathbf{U}_0 = 0. \tag{2}$$

In the variables stream function (ψ)—vorticity (ω) and the polar coordinates r, φ the system of equations (2) has the dimensionless form:

$$\mathrm{Re}^*(\mathbf{U}_0\cdot\nabla)\omega + \mathrm{Gr}^*\left(r\frac{\partial\theta}{\partial r}\cos(\alpha+\varphi) - \frac{\partial\theta}{\partial\varphi}\sin(\alpha+\varphi)\right) = \mathrm{Da}\,\Delta_{r\varphi}\omega - r\omega,$$
$$\mathrm{Re}^*(\mathbf{U}_0\cdot\nabla)\theta = \frac{6}{5}\frac{\mathrm{Da}}{\mathrm{Pr}}\Delta_{r\varphi}\theta,\quad u_0 = \frac{1}{r}\frac{\partial\psi}{\partial\varphi},\quad v_0 = -\frac{\partial\psi}{\partial r},$$
$$\Delta_{r\varphi}\psi = r\omega,\quad \Delta_{r\varphi} = \frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial\varphi^2}. \tag{3}$$

The variable r has been divided by the cylinder radius R ($R \gg h$), the velocity components u_0, v_0 by $\bar{U}_0$ ($\bar{U}_0 = 1.5\bar{U}$, $\bar{U}$ is the flow-average velocity), and ψ and ω by $\bar{U}_0 R$ and $\bar{U}_0/R$; the dimensionless temperature $\theta = (T - T_\infty)/(T_0 - T_\infty)$.

We will consider the symmetric flow past a cylinder when the free-stream velocity and gravity vectors act in opposite ($\alpha = 90°$) and the same ($\alpha = -90°$) directions. The boundary conditions for Eqs. (3) are: $\theta(1, \varphi) = 1$, $\theta(\infty, \varphi) = 0$, $u_0(1, \varphi) = 0$, $v_0(1, \varphi) = 0$, $u_0(\infty, \varphi) = \cos\varphi$, $v_0(\infty, \varphi) = -\sin\varphi$.

The dimensionless numbers: $\mathrm{Re}^* = (\bar{U}h^2)/(10\nu R)$ – the reduced Reynolds number, $\mathrm{Gr}^* = (|g|\beta(T_0 - T_\infty)h^2R)/(12\nu\bar{U})$ – the reduced Grashof number, and $\mathrm{Da} = h^2/(12R^2)$ – the Darcy number for the Hele–Shaw cell are the internal parameters of the problem. The external parameters are the Re and G numbers calculated from the thickness of the cell and the flow-average velocity: $\mathrm{Re} = (\bar{U}h)/\nu$, $\mathrm{Gr} = (|g|\beta(T_0 - T_\infty)h^3)/(\bar{U}\nu)$, which must correspond to the laminar flow range.

The system of equations (3) was solved by a finite-difference method. We used a conservative upwind difference scheme [7] with first-order-accurate approximation of the convective term and second-order-accurate approximation of the second derivatives. The system of nonlinear algebraic equations obtained was solved by the Gauss–Seidel iteration method. The solution was found on grids nonuniform with respect to the r coordinate (with closer spacing near the surface of the cylinder) and uniform with respect to the φ coordinate. The boundary conditions at $r = \infty$ were imposed at $r = 15$. In this case the effect of the outer boundary on the flow near the lateral surface of the cylinder was insignificant.

The principal integral characteristic of the thermal problem is the average Nusselt number on the cylinder wall

$$\mathrm{Nu} = -\frac{1}{\pi}\int_0^\pi \frac{\partial T}{\partial r}(1, \varphi)\,d\varphi.$$

For calculating Nu we used a 21 × 21 grid with a minimum step of 0.001 near the cylinder wall. On transition to a denser 61 × 61 grid Nu changed by not more than 3-5% for all the values of the problem parameters on the intervals mentioned below. For obtaining the steady flow pattern near the cylinder we also used a 61 × 61 grid.

The multivariant calculations were carried out for values of the parameters Re*, Gr*, and Da on the intervals 0.1-10, 0-10, and 0.0001-0.001, respectively. The Prandtl number was varied from 1 to 10. Detailed calculations with a step Re* = 0.2 enabled us to determine the value Re_0^* at which the flow separates from the cylinder wall when Gr* = 0. Irrespective of Da, $\mathrm{Re}_0^* \approx 1.5$. With increase in Gr* on the interval $\mathrm{Re}_0^* < \mathrm{Re}^* < 3$ unseparated

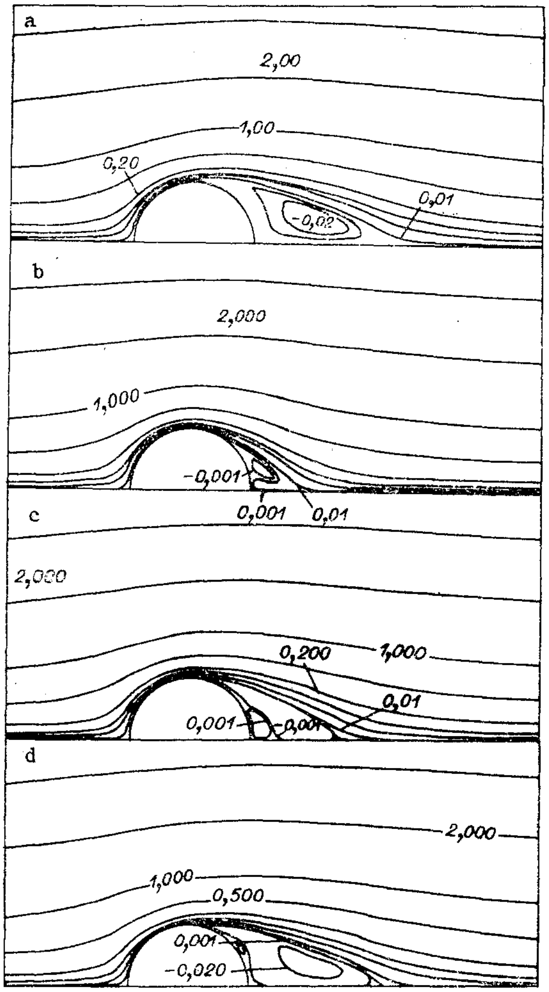

Fig. 1

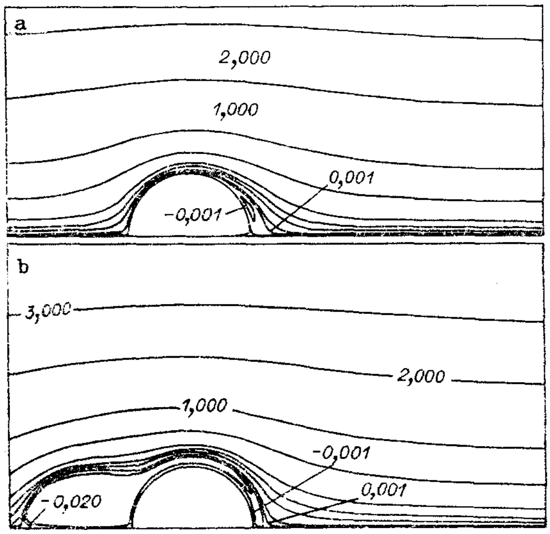

Fig. 2

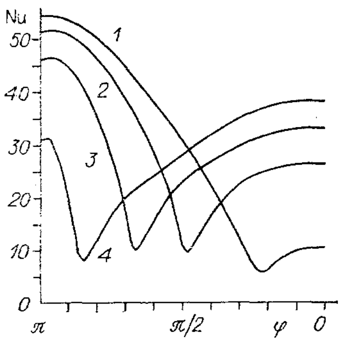

Fig. 3

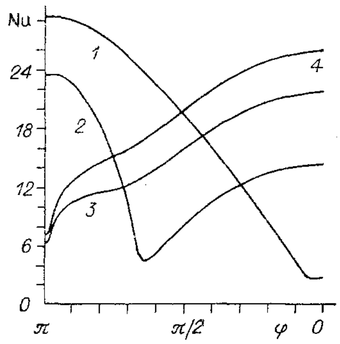

Fig. 4

flow past the cylinder is realized when $Re^* > Re_0^*$. When $Re^* > 3$ the flow is separated at all values of Gr^*.

When $\alpha = 90°$ the interaction of the inertia and buoyancy forces determines the formation of the following types of separation zones, depending on the number of zeros of the vorticity on the cylinder wall (the streamline configurations in Fig. 1 and Fig. 2 correspond to $Da = 0.001$): (a) one zero – flow separation in the absence of buoyancy forces when $Gr^* = 0$ (Fig. 1a, $Re^* = 10$); (b) two zeros – flow past cylinder with reattachment of the separated flow to the cylinder wall (Fig. 1b, $Re^* = 3$, $Gr^* = 1$) and with the formation of two new vortices (Fig. 1c, $Re^* = 6$, $Gr^* = 1$); (c) three zeros – initial stage of formation of vortices due to fluid convection (Fig. 1d, $Re^* = 10$, $Gr^* = 1$).

When $\alpha = -90°$ the flow past a cylinder with codirectional gravity and free-stream velocity vectors is characterized by the formation of stagnant zones near the surface of the cylinder when $Re^* < Re_0^*$ as a result of the convective motion of the fluid. A stagnant zone is formed on the rear (relative to the free-stream direction) surface of the cylinder (Fig. 2a, $Re^* = 1$, $Gr^* = 1$). As Gr^* increases, the flow separation point moves upstream and an extensive stagnation zone is formed in front of the cylinder (Fig. 2b, $Re^* = 0.3$, $Gr^* = 10$). As Re^* increases, the stagnant zone is displaced towards the rear of the cylinder.

On the cylinder wall when $\alpha = -90°$ Nu is almost independent of Gr^* for $Re^* > 1$ (as Gr^* varies from 0 to 10, Nu decreases by 5-7%). In this case the fall in heat transfer in front of the cylinder as Gr^* increases is compensated by the increased heat transfer in the area of the stagnant zone (Fig. 3, $Re^* = 3$, $Da = 0.001$, the local Nu distributions on the cylinder wall 1-4 correspond to $Gr^* = 1$, 4, 7, and 10).

In Fig. 4 we have plotted the dependence of the local Nusselt number on the cylinder wall on Gr^* for $\alpha = -90°$, $Da = 0.001$, and $Re^* = 0.3$. The curves 1-4 correspond to $Gr^* = 0$, 1, 4, and 7. When $Re^* < 1$ the heat transfer from the lateral surface of the cylinder decreases with increase in Gr^* until convective motion of the fluid counter to the free stream develops along the entire surface of the cylinder. With further increase in Gr^* the heat transfer increases (Fig. 4, curves 3 and 4).

When the free-stream velocity and gravity vectors act in different directions ($\alpha = 90°$), the dependence of Nu/Nu_0 (Nu_0 is the average Nusselt number when $Gr^* = 0$) on Gr^* has the form:

$$Nu/Nu_0 = 1 + 0{,}08 Gr^{*0{,}7} \tag{4}$$

(Nu/Nu_0 does not depend on Da).

The criterial relation for Nu_0 when $Re < 1000$ ($Re^* < 300 Da^{0.5}$) can be written as

$$Nu_0 = c\, Pe^{*0{,}4} Da^{-0{,}5}, \tag{5}$$

where $Pe^* = Re^* Pr$ is the reduced Péclet number. When $Re^* \leq 1$, $c \simeq 0.47$, and when $Re^* \geq 2$, $c \simeq 0.44$. The calculated values of the average Nusselt number and those found from expressions (4) and (5) coincide to within 7%.

Thus, for parallel gravity and free-stream velocity vectors the effect of the buoyancy forces on the heat transfer from the lateral surface of a cylinder in a flat channel is manifested only when the vectors act in opposite directions. The generalizing relations (4) and (5), obtained on the basis of a large number of calculations in accordance with the proposed flow model, which takes inertial effects into account, make it possible to estimate the total heat transfer from the lateral surface of a heat-release element in a flat channel in the presence of moderate temperature differences in the fluid.

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